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A Benders decomposition approach to transmission expansion planning considering energy storage

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Abstract

We describe an electricity transmission network expansion and energy storage planning model (TESP) that determines the location and capacity of energy storage systems (ESS) in the network for the purposes of demand shifting and transmission upgrade deferral. This problem is significantly harder than the standard network expansion models that are typically considered literature as the benefit of storage can only be understood by including multiple time periods in the model. The addition of the time dimension leads to much larger mixed integer linear programming problems. We address this increase in size and complexity of the optimization problem by developing a Benders decomposition approach for the TESP. The model is tested against the well known Garver's 6-bus, IEEE 25-bus, and Brazilian 46-bus test systems under two different demand scenarios; the first is characterized by a short period of peak demand, the second by a long period. Benders decomposition is shown to be an effective means to render the problem more tractable when compared to the standard mixed integer linear programming approach. We find that installation of ESS is an effective means of transmission upgrade deferral. However storage is unlikely to be installed where circuit installation is of comparatively low cost.

Keywords: power transmission, energy storage, optimization

1. Introduction

Electrical transmission network expansion planning (TEP) is a challenging optimization problem with the objective of minimizing investment and operational costs of the expanded transmission network infrastructure (pylons, lines, transformers, etc.) while meeting capacity, demand, security, geographical, or environmental constraints [1].

The integration of renewable energy generation into the network, in particular variable forms of generation such as wind and solar, present a significant challenge for planners and as a result there is a renewed interest in electricity network planning problems [2].

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One strategy to address these challenges is to use energy storage systems (ESS) to smooth the supply and match the demand in the network. The most common form of storage for large amounts of energy is hydro, which while relatively cheap, is limited by both geography and climate. Other forms of storage such as batteries or compressed air may also be feasible but are currently significantly more expensive.

In this paper, we build upon the TEP to develop a transmission network expansion and energy storage planning (TESP) model that determines the location of ESS in the network, and how this storage might be used for the dual purposes of demand shifting and transmission upgrade deferral. The difference between the two is somewhat subtle. Demand shifting involves storing energy generated in one time period in order to match the demand in a subsequent period. Transmission upgrade deferral requires storing energy close to sources of generation or demand and moving it at a steady rate over time to avoid the need for larger capacity or additional transmission lines.

Energy storage is not the only means to facilitate network upgrade deferral. One alternative is the installation of distributed generation (DG) which may be operated with similar peak cutting effects to storage. A multi-objective model for distribution network planning (DEP) that considers DG as an alternative to circuit reinforcements is given in [3].

Transmission expansion planning problems are frequently modeled as mixed integer nonlinear programs (MINLP), or in an equivalent disjunctive mixed integer program (MIP) form. The standard models and test systems, as well as a worked example, are detailed in [4].

Small, linear network expansion models are quite readily solved to optimality using a modern commercial solver such as IBM ILOG CPLEX. Numerous specialist solution methods have been developed to address the difficulty typically experienced while solving large instances, including Projection-Adapted Cross Entropy [5], branch and bound with a GRASP meta-heuristic [6], heuristic methods [7], and evolutionary procedures such as genetic algorithms [8] and particle swarm optimization [9]. Many of these approaches are examined in more detail in [10].

The TEP problems often may be decomposed into investment and operation subproblems. Benders decomposition with investment subproblems with continuous or discrete decision variables, and transportation and DC approximation operation subproblems are compared in [11]. Additional constraints on new paths, and fencing constraints added to the investment subproblem are shown to reduce the number of iterations required substantially [12]. Gomory cuts have been added at each iteration to solve the linear disjunctive MIP model [13]. More recently a probabilistic model considering uncertain generation (wind) and variable load has been proposed [14]. Local branching is used to accelerate the Benders decomposition of a TEP problem in [15], however the authors do not compare the technique with using solver callbacks and a 'one tree' master problem.

A linear direct current (DC) approximation is often sufficient for the purposes of long term transmission expansion planning given the computational complexity of modeling high-voltage three-phase alternating current (AC) power flows, and is the approach we use in this paper. However, some recent research has also considered AC power models, including step by step development of an expansion plan using an interior point method and a constructive heuristic algorithm [16], and the development of a binary linear AC model of comparable efficiency to the more traditional DC approximation model [17].

The integration of ESS into TEP and related problems, such as distribution network expansion planning (DEP), is an emerging area of research.

As we demonstrate in this paper, the expansion plan produced when considering energy storage depends upon the temporal location of the peak load and the distribution of load around the peak. Therefore, the transmission expansion planning problem becomes significantly more complicated as the model has to include multiple time periods. The typical approach for standard TEP, by contrast, only models the peak supply and demand and thus peak load on the network links. Some early work on incorporating storage into TEP also ignores the time dimension so that the storage facilities essentially behave like an alternative type of generator [18].

Clack et al. compare a pair of linear programming models [19] to design a high voltage direct current (HVDC) transmission network which takes in to account variable and dispatchable generation as well as energy storage. They find that the first load matching model installs a higher proportion of variable generation and is less computationally demanding than the second cost minimizing model. The load matching model installs 32 GW of energy storage, but none is installed by the cost minimizing model due to its high cost. This high level network design formulation differs from MIP formulations such as the model presented in this paper in a number of ways. For example, it determines the total required capacity along a right of way, but not the number of discrete new lines to install.

The location and sizing of storage has also been considered in distribution networks. Approaches here include techniques such as the optimization of a multi-period design problem using a modified particle swarm optimization PSO [20], and planning a low voltage distribution network with high solar PV penetration using a genetic algorithm in combination with simulated annealing [21].

The cost per MW of long term (~4 hours) energy storage, such as pumped hydro or flow batteries, was estimated to be AUD$810,451 (US$842,058) in 2012 [22, p.43]. Costs of this magnitude prohibit the installation of storage in the test systems presented in this paper even where the time value of money is considered, and therefore coefficients of convenience are used to demonstrate the viability of the solution method. However, it should be noted that rapid developments in ESS technology are reducing the cost of storage and this may no longer be necessary in future.

The rest of this paper is organized as follows. A MIP formulation of the TEP with storage model is given in Section 2, Sections 3, 4 and 5 provide three case studies in which we test the model on the Garver's 6-bus, IEEE 25-bus and Brazilian 46-bus test systems. In Section 6 we discuss our experimental results and we conclude in Section 7.

2. Mathematical model

The TEP problem is typically solved to determine the minimum cost expansion plan that satisfies some peak system demand. This approach was used by Hu et. al [18] to develop an extension to the traditional disjunctive TEP formulation that considers the location of ESS, and by Zhang et. al [23] whose linear mixed integer model also considers line losses.

In [18] an upper bound on the total investment amount is established by solving the TEP problem without storage. An upper bound on the number of ESS to consider is then set to a value appropriate for the system size. This problem is then solved iteratively, decrementing the number of ESS each iteration if resulting expansion plan differs from the plan without storage, otherwise terminating the algorithm. The set of expansion plans are then analyzed.

These peak demand approaches facilitate transmission upgrade deferral by specifying the rated power of ESS within the network. However, they do not demonstrate that sufficient generation or transmission capacity exists in any prior time period to operate the energy storage. Taking this into account, our approach is significantly different. We introduce discrete time periods with variable demand into the model in order to operate the ESS like a rechargeable battery, that is, alternately as an energy demand centre or an energy generator. This ensures the generated expansion plan is feasible for the given operating conditions.

As with the standard TEP models, the objective of our TESP model is to minimize the investment costs incurred by expanding the transmission network, and to minimize a penalty for load curtailment at each demand node which is often used to represent operational costs. The model allows for the installation of discrete new or reinforcing circuits on a right of way. It determines the location and sizing of continuous capacity storage within the network. ESS will only be installed if it is more economical than to install one or more new circuits or to curtail load.

The model implements cyclic discrete time which requires the state of the storage at the last time period to be identical to the initial storage state. The duration of each timestep is expected to be in the order of 30 minutes to an hour, although longer timesteps commensurate with modeled planning horizon are possible. Intra time period demand and generation are assumed to be constant, however generation is re-dispatchable and demand may vary over time.

Since our modelling of time differs from some of the other approaches in the literature it is worth considering some of the alternatives to our approach:

- Single time period: makes it impossible to estimate the energy capacity of the storage. Furthermore, reducing power flow into a bus by using storage in one period means that at another time period the power flow must be increased to enable recharging. Hence, a single time period model can at best give a very crude approximation to the effect of allowing storage to be installed. Of course in the case where the supply and demand is constant, the single time period model would be reasonable to use. However, in such cases storage adds no value to the network, and in practice the demand is never constant.

- Non-cyclic time: instead of considering a single day or single week with a requirement that storage starts and finishes at the same level, it is possible to simply consider a period of time without cyclic constraints. However, this introduces edge effects where the storage may be run down at the end of the planning period or perhaps assumed to be full at the start in order to reduce the load on the network. In order to eliminate the impact of the edge effects on the expansion planning decisions, a much longer time period has to be considered.
- Multiple (cyclic) periods: A multi-scenario approach could be allowed for where not just a single pattern of demand is considered, but a number of representative cyclic patterns (eg. to capture a summer and winter pattern of usage). This does not significantly change the models and in fact our Benders decomposition approach would easily extend to this. However, including this makes both the presentation of the mathematical model more complex and may significantly increase the computational time required. As the current models already take significant amounts of time to run, this extension has not been included here.

Power flows are modeled using a DC approximation [24, p.36], with subsidiary decisions to determine the phase angles at each bus, network flows, and the amount of energy stored in ESS for each time period.

Transmission expansion planning is considered static if the planner is concerned only with determining a final network plan, whereas the planning is dynamic if one or more intermediate plans, perhaps over multiple time periods are determined. Although our model includes time periods, the planning is static, i.e. we determine only the final plan.

The following notation will be used throughout this paper:

Sets

Γ	the set of indices for buses;
Ω_0	the set of rights of way for existing circuits;
Ω_c	the set of rights of way for candidate circuits;
Ψ	the set of time periods $\{1, 2, \ldots, T\}$;

Parameters

α_{tk} cost of curtailment at time t at bus k;
b_k cost of installing storage at bus k;
c_{ij} cost of installing a circuit on right of way ij;
d_{tk} demand at time t at bus k;
$\bar{f}_{ij}$ maximum possible power flow on right of way ij;
$\bar{g}_k$ maximum possible generation at bus k;
γ_{ij} susceptance of circuits installed on right of way ij;
M_{ij} the disjunctive parameter for right of way ij
n^0_{ij} number of existing circuits on right of way ij;
$\bar{n}_{ij}$ maximum number of installable circuits on right of way ij;
$\bar{x}_k$ maximum installable storage capacity at bus k;

Decision variables

β_{tk} power flow to storage at bus k at time t;
g_{tk} generation at time t at bus k;
f^0_{tij} power flow for existing circuits at time t on right of way ij;
f^p_{tij} power flow for the p^{th} candidate circuit at time t on right of way ij;
l_{tk} level of storage at bus k at time t;
r_{tk} demand curtailment at time t at bus k ;
θ_{tk} phase angle at time t at bus k;
x_k storage capacity installed at bus k;
y^p_{ij} binary variable denoting installation of the p^{th} candidate circuit on right of way ij;

As described in Section 1, it is often convenient to reformulate classical nonlinear DC approximation model in an equivalent disjunctive mixed integer linear programming form. In [25] we introduced a model that builds upon the disjunctive model given in [4]. The formulation of this model differs entirely from its foundation as it considers discrete cyclic time periods, demand that varies over time, and the selection and location of ESS:

The objective is to minimize the function

$$z = \sum_{(i,j)} c_{ij} y^p_{ij} + \sum_{k\in\Gamma} b_k x_k + \sum_{t\in\Psi}\sum_{k\in\Gamma} \alpha_{tk} r_{tk} \tag{2.1}$$

where c_{ij} is cost of installing a line on right of way ij and y^p_{ij} is a binary variable denoting the installation of the p^{th} candidate line on ij. It is assumed that the variable operating cost of ESS is negligible in relative terms, and only the fixed costs b_k of installing x_k MW of storage at bus k are included in the objective function. At each bus, r_{tk} load may be curtailed in each time period t at a cost of αtk.

The technical constraints that define the expansion plan are outlined below:

Nodal balance and power flow

$$\zeta + g_{tk} + r_{tk} - \beta_{tk} = d_{tk} \qquad \forall\, t \in \Psi,\ \forall\, k \in \Gamma \tag{2.2}$$

where

$$\zeta = \sum_{(i,k)\in\Omega_0} f^0_{tik} - \sum_{(k,j)\in\Omega_0} f^0_{tkj} + \sum_{p=1}^{\bar{n}_{ij}} \sum_{(i,k)\in\Omega_c} f^p_{tik} - \sum_{p=1}^{\bar{n}_{ij}} \sum_{(k,j)\in\Omega_c} f^p_{tkj} \tag{2.3}$$

Nodal balance i.e. Kirchhoff's current law is ensured at each time period by constraint (2.2).

Power flows are modeled using a DC approximation resulting in subsidiary decisions to determine the phase angle at each bus:

$$f^0_{tij} - \gamma_{ij} n^0_{ij} \left(\theta_{ti} - \theta_{tj}\right) = 0 \quad \forall\, t \in \Psi, \forall\ (i,j) \in \Omega_0 \tag{2.4}$$

$$\left|f^p_{tij} - \gamma_{ij} \left(\theta_{ti} - \theta_{tj}\right)\right| \leq M_{ij}(1 - y^p_{ij}) \quad \forall\, t \in \Psi,\ \forall\ (i,j) \in \Omega_c,\ \forall\, p \in \{1 \ldots \bar{n}_{ij}\} \tag{2.5}$$

Kirchhoff's voltage law is implemented for existing circuits by (2.4), and for candidate circuits by (2.5). The disjunctive parameter M_{ij} must be sufficiently large number so that the difference in phase angles of buses i and j is not artificially limited. A procedure for calculating minimum values of M_{ij} is given in [13].

$$\left|f^0_{tij}\right| \leq n^0_{ij} \bar{f}_{ij} \quad \forall\, t \in \Psi,\ \forall\ (i,j) \in \Omega_0 \tag{2.6}$$

$$\left|f^p_{tij}\right| \leq y^p_{ij} \bar{f}_{ij} \quad \forall\, t \in \Psi,\ \forall\ (i,j) \in \Omega_c,\ \forall\, p \in \{1 \ldots \bar{n}_{ij}\} \tag{2.7}$$

Nominal thermal limits are enforced on existing and candidate circuits by constraint (2.6) and constraint (2.7) respectively.

Storage level and charge/discharge limits

$$l_{1k} = l_{Tk} + \beta_{1k} \quad \forall\, k \in \Gamma \tag{2.8}$$

$$l_{tk} = l_{t-1,k} + \beta_{tk} \quad \forall\, t \in \Psi,\ \forall\, k \in \Gamma \tag{2.9}$$

As we are operating the storage over some typical demand scenario, say a day, the set of time periods Ψ is assumed to be cyclic. Thus, the level of the storage at the last time period is required to be identical to the initial storage state. This requirement is implemented by the "wrap around" constraint (2.8). For all other time periods the storage level is given by (2.9).

$$0 \leq l_{tk} \leq x_k \quad \forall\, t \in \Psi,\ \forall\, k \in \Gamma \tag{2.10}$$

$$0 \leq x_k \leq \bar{x}_k \quad \forall\, k \in \Gamma \tag{2.11}$$

Constraint (2.10) ensures the stored energy does not exceed the installed capacity, while constraint (2.11) establishes bounds on this capacity.

In this formulation, power flows into and out of ESS are limited only by the capacity and level of storage, as well as the capacity of connected transmission lines. Subject to these limitations, it is theoretically possible that the storage completely charge or discharge within a single time period. Furthermore, the model assumes 100% efficiency for storage and losses are not considered.

Generation bounds

$$0 \leq g_{tk} \leq \bar{g}_k \quad \forall\, t \in \Psi,\ \forall\, k \in \Gamma \tag{2.12}$$

Generator re-dispatch is permitted within the bounds imposed by (2.12).

Curtailment bounds

$$0 \leq r_{tk} \leq d_{tk} \quad \forall\, t \in \Psi,\ \forall\, k \in \Gamma \tag{2.13}$$

Curtailment at any node during a given time period cannot exceed the demand at that node during the same time period.

Symmetry breaking constraints

$$y_{ij}^p \geq y_{ij}^{p+1} \quad \forall\ (i, j) \in \Omega_c,\ \forall\, p \in \{1 \dots \bar{n}_{ij} - 1\} \tag{2.14}$$

The lexicographical constraint (2.14) eliminates the symmetry introduced by the inclusion of the binary decision variables by mandating the order of installation of parallel circuits be arbitrary.

Other

$$y_{ij}^p \in \{0, 1\} \tag{2.15}$$

$$f_{tij}^0, f_{tij}^p, \beta_{tk}, \theta_{tk} \text{ unbounded} \tag{2.16}$$

Smaller instances of this monolithic TEP model may be solved using a commercial solver, however the problem may become intractable as the number of dimensions increases. To remedy this, it is frequently required to decompose the problem.

Benders decomposition divides the problem into a master problem containing the integer variables (and optionally some of the continuous variables), and a subproblem containing the continuous variables [26].

Benders decomposition has been applied to a wide range of problems including unit commitment [27], aircraft routing and crew scheduling [28], and the fixed charge network design problem [29].

The TEP model can be decomposed into master and dual subproblems.

Minimize:

$$z = \sum_{(i,j)} c_{ij} y_{ij}^p + v \tag{2.17}$$

Subject to:

$$y_{ij}^p \geq y_{ij}^{p+1} \quad \forall\ (i,j) \in \Omega_c,\ \forall\ p \in \{1 \ldots \bar{n}_{ij} - 1\} \tag{2.18}$$

$$v \geq 0 \tag{2.19}$$

$$y_{ij}^p \in \{0,1\} \tag{2.20}$$

The objective of the master problem is to minimize the cost of investment in transmission lines, as well as to minimize the estimated objective function values of the subproblem v. Only the lexicographical symmetry breaking constraints are retained as the operational constraints now appear in the subproblem.

Let the dual variables $\pi_{d_{tk}}$ be associated with constraint (2.2), $\pi_{\gamma_{tij}}$ with constraint (2.4), $\pi_{\gamma_{tij}^{+p}}$ and $\pi_{\gamma_{tij}^{-p}}$ with constraint (2.5), $\pi_{f_{tij}^{+0}}$ and $\pi_{f_{tij}^{-0}}$ with (2.6), and $\pi_{f_{tij}^{+p}}$ and $\pi_{f_{tij}^{-p}}$ with (2.7). The dual variables $\pi_{s_{tk}}$ are associated with constraints(2.8) and (2.9), and $\pi_{\bar{l}_k}$ with (2.10). Finally, let the dual variables $\pi_{g_{tk}}$, $\pi_{r_{tk}}$, and π_{xk} be associated with the bounds (2.11 - 2.13) respectively.

The dual of the subproblem can be formulated as follows:

Maximize:

$$\begin{aligned} v = &\sum_{t\in\Psi}\sum_{k\in\Gamma} d_{tk}\pi_{d_{tk}} + \sum_{t\in\Psi}\sum_{(i,j)\in\Omega_0} \left[\pi_{f_{tij}^{+0}} n_{ij}^0 \bar{f}_{ij} + \pi_{f_{tij}^{-0}} n_{ij}^0 \bar{f}_{ij}\right] + \sum_{t\in\Psi}\sum_{(i,j)\in\Omega_c} \left[\pi_{f_{tij}^{+p}} \hat{y}_{ij}^p \bar{f}_{ij} + \pi_{f_{tij}^{-p}} \hat{y}_{ij}^p \bar{f}_{ij}\right] \\ &+ \sum_{t\in\Psi}\sum_{(i,j)\in\Omega_c} \left(\pi_{\gamma_{tij}^{+p}} + \pi_{\gamma_{tij}^{-p}}\right)\left(M_{ij}(1-\hat{y}_{ij}^p)\right) + \sum_{t\in\Psi}\sum_{k\in\Gamma} \bar{g}_k \pi_{g_{tk}} + \sum_{t\in\Psi}\sum_{k\in\Gamma} d_{tk}\pi_{r_{tk}} + \sum_{k\in\Gamma} \bar{x}_k \pi_{x_k} \end{aligned} \tag{2.21}$$

Subject to:

$$\pi_{d_{tj}} - \pi_{d_{ti}} + \pi_{f_{tij}^{+0}} - \pi_{f_{tij}^{-0}} + \pi_{\gamma_{tij}} = 0 \qquad \forall\ t \in \Psi,\ \forall\ (i,j) \in \Omega_0 \tag{2.22}$$

$$\pi_{d_{tj}} - \pi_{d_{ti}} + \pi_{f_{tij}^{+p}} - \pi_{f_{tij}^{-p}} + \pi_{\gamma_{tij}^{+p}} - \pi_{\gamma_{tij}^{-p}} = 0 \qquad \forall\ t \in \Psi,\ \forall\ (i,j) \in \Omega_c,\ \forall\ p \in P \tag{2.23}$$

$$\begin{aligned} &\sum_{(i,k)\in\Omega_0} \gamma_{ik} n_{ik}^0 \pi_{\gamma_{tik}} - \sum_{(k,j)\in\Omega_0} \gamma_{kj} n_{ik}^0 \pi_{\gamma_{tkj}} + \sum_{p\in P}\sum_{(i,k)\in\Omega_c} \gamma_{kj}\pi_{\gamma_{tik}^{+p}} - \gamma_{kj}\pi_{\gamma_{tik}^{-p}} \\ &\qquad + \sum_{p\in P}\sum_{(k,j)\in\Omega_c} \gamma_{kj}\pi_{\gamma_{tkj}^{-p}} - \gamma_{kj}\pi_{\gamma_{tkj}^{+p}} = 0 \qquad \forall\ t \in \Psi,\ \forall\ k \in \Gamma \end{aligned} \tag{2.24}$$

$$\pi_{s_{tk}} - \pi_{s_{t+1,k}} + \pi_{\bar{l}_k} \leq 0 \qquad \forall\ t > 1 \in \Psi,\ \forall\ k \in \Gamma \tag{2.25}$$

$$\pi_{s_{Tk}} - \pi_{s_{2k}} + \pi_{\bar{l}_k} \leq 0 \qquad \forall\ t = 1,\ k \in \Gamma \tag{2.26}$$

$$-\pi_{d_{tk}} - \pi_{s_{tk}} = 0 \qquad \forall\, t \in \Psi,\ \forall\, k \in \Gamma \tag{2.27}$$

$$\pi_{d_{tk}} + \pi_{g_{tk}} \leq 0 \qquad \forall\, t \in \Psi,\ \forall\, k \in \Gamma \tag{2.28}$$

$$\pi_{x_k} - \pi_{\bar{l}_{tk}} \leq b_k \qquad \forall\, t \in \Psi,\ \forall\, k \in \Gamma \tag{2.29}$$

$$\pi_{d_{tk}} + \pi_{r_{tk}} \leq \alpha_{tk} \qquad \forall\, t \in \Psi,\ \forall\, k \in \Gamma \tag{2.30}$$

$$\pi_{f^{+0}_{tij}}, \pi_{f^{-0}_{tij}}, \pi_{f^{+p}_{tij}}, \pi_{f^{-p}_{tij}}, \pi_{\gamma^{+p}_{tij}}, \pi_{\gamma^{-p}_{tij}}, \pi_{g_{tk}}, \pi_{r_{tk}}, \pi_{\bar{l}_{tk}}, \pi_{x_{tk}} \leq 0 \text{ and } \pi_{d_{tk}}, \pi_{\gamma_{tij}}, \pi_{s_{tk}} \text{ unbounded} \tag{2.31}$$

As load curtailment is permitted at any node, the subproblem remains bounded for any feasible solution to the master problem. This means that we need only consider the optimality cut:

$$v - \sum_{t\Psi} \sum_{(i,j)\in\Omega_c} \left[\pi_{f^{+p}_{tij}} y^p_{ij} \bar{f_{ij}} + \pi_{f^{-p}_{tij}} y^p_{ij} \bar{f_{ij}}\right] - \sum_{t\in\Psi} \sum_{(i,j)\in\Omega_c} \left(\pi_{\gamma^{+p}_{tij}} + \pi_{\gamma^{-p}_{tij}}\right) \left(M_{ij}(1 - y^p_{ij})\right) \geq$$
$$\sum_{t\in\Psi}\sum_{k\in\Gamma} d_{tk}\pi_{d_{tk}} + \sum_{t\in\Psi}\sum_{(i,j)\in\Omega_0} \left[\pi_{f^{+0}_{tij}} n^0_{ij} \bar{f_{ij}} + \pi_{f^{-0}_{tij}} n^0_{ij} \bar{f_{ij}}\right] + \sum_{t\in\Psi}\sum_{k\in\Gamma} \bar{g_k}\pi_{g_{tk}} + \sum_{t\in\Psi}\sum_{k\in\Gamma} d_{tk}\pi_{r_{tk}} + \sum_{k\in\Gamma} \bar{x_k}\pi_{x_k} \tag{2.32}$$

The model is implemented using the Python library for IBM ILOG CPLEX 12.6, and lazy constraint callbacks are used to solve the subproblem and separate the cuts. By default preprocessing is disabled, and the branch and cut is single threaded, although the LP solver may take advantage of multi-threading when solving the subproblems.

3. Case study: Garver's 6-bus network

We compare the performance of the monolithic and decomposed versions of the model using Garver's 6-bus test system. In this test system there are 6 buses, 15 rights of way, and matching generation and demand of 750MW. Fig. 1 and Fig. 2 show the initial network topology and optimal transmission expansion plan without considering ESS respectively. Four new circuits are installed on right of way 2−6, two new circuits on right of way 4−6, and one reinforcing circuit is installed on right of way 3−5 at a total investment cost of US$200,000. The expanded transmission network is capable of satisfying peak load of 760MW without load curtailment.

It is typical to consider only peak demand in the network for transmission expansion planning. However, a key assumption in our model is that an installed ESS will store energy during periods of low demand and export energy during periods of high demand. In the following case studies we consider two different demand scenarios over a period of 24 hours with a 30 minute time step: a short peak scenario and a long peak scenario. Each scenario is inspired by a real world demand time series, and is presented in a simplified form to assist in replicating the results.

For each scenario, demand in the entire network over time is shown in Fig. 3. The short peak scenario is characterized by low demand of 456MW over the first 6 hours, building steadily over the next 10 hours to a peak of 760MW, before decreasing again to a period of low demand. Over the 24 hour time period this scenario has mean demand of 557MW. The long peak scenario is likewise characterized by low demand over the first 6 hours. Demand then steeply increases to a peak of 760MW where it remains constant for the next 10 hours, before moderating at the same rate observed in the short peak scenario to a period of low demand late in the day. Mean demand over 24 hours is 605MW.

We re-scale demand at all buses using a single scenario, but multiple scenarios may also be used if desired. For example, demand at bus 2 is re-scaled at each timestep to be a proportion of the maximum demand of 240MW. Demand at bus 1, etc. could be similarly re-scaled, or a different scenario applied.

We impose an upper bound of 500 MWh on the installation of ESS at all 6 buses. However, this is not a requirement of the formulation and the modeler is free to determine which buses are candidates for ESS installation, as well as the maximum capacity of any installed ESS.

In order to demonstrate the use of the model, the maximum storage cost coefficient that ensured storage was installed was found for each demand scenario. Results are given in Table 1.

For the short peak scenario 80 MWh of storage is installed at bus 2 at a cost of \$370/MWh or below. This allows one circuit on right of way 2-6 to be omitted from the expansion plan which results in a modest improvement in the objective function value. The long peak scenario requires an additional 333 MWh of storage at a maximum cost of \$70/MWh. The total cost savings under this scenario are similarly modest. The optimal expansion plan for the short peak scenario with storage priced at \$70/MWh is given to enable direct comparison.

As the same set of new and reinforcing circuits are installed for each scenario, the viability of deploying ESS as a means of transmission upgrade deferral is at least in part dependent on the nature of demand during the time period in which the storage is operated, but the most significant factor is cost. The required expansion plan for the short peak scenario is shown in Fig. 4.

A total of 13 optimality cuts are added for the short peak scenario with a storage cost of \$70 /MWh while 12 optimality cuts are added for the long peak scenario. When the storage cost is \$370 /MWh 18 optimality cuts are required for the short peak scenario. Given the trivial nature of the problems, the wall time for each scenario is only a couple of seconds.

This case study simply illustrates that the consideration of storage in the TEP produces different solutions than TEP without storage. Furthermore the characteristics of the demand variability has a significant impact on the solution cost.

4. Case study: IEEE 25-bus network

The IEEE 25-bus test system extends the well know IEEE 24-bus reliability network. The system has 25 buses, 36 rights of way, and total demand of 2750 MW. The tabulated data and a diagram are given in [30]. Permitting a maximum of 4 new or reinforcing circuits on each right of way, the optimal expansion plan without storage has a cost of US$107.7 million. One circuit is installed on rights of way 1-2 and 7-13, two circuits are in installed on rights of way 12-14, 13-18 and 24-25, three on rights of way 8-22 and 12-23, and four on right of way 13-20.

In this case study we use the same short peak and long peak scenarios used in Section 3, and demand is similarly re-scaled at each bus. To demonstrate the use of the model this larger problem we use an ESS cost coefficient of 1.0, which given the 30 minute time-step is equivalent to $2000/MWh.

The model is solved using IBM ILOG CPLEX 12.6 on a cluster node with 4 processors and 16GB of RAM. The performance of the Benders decomposition is compared to solving the monolithic formulation with CPLEX configured to use a single thread (denoted CPLEX 1), and CPLEX configured deterministic parallelism using up to 16 threads (denoted CPLEX 16). Numerical results are given in Table 2.

For the short peak scenario a total of 12 new circuits on 5 rights of way are combined with 1598 MWh of energy storage at a cost of US$32 million. The optimal expansion plan for the long peak scenario costs US$43.8 million and requires that 11 new circuits on 6 rights of way be combined with 2619 MWh of energy storage. In each case a significant cost saving is achieved because the installation of expensive transmission lines is deferred due to the availability of comparatively cheap storage.

Where storage is not considered the model need only solve a single, peak time period. With the use of contemporary solvers and hardware the time required to determine the optimal solution of the monolithic formulation is only a few seconds. The introduction of discrete time into the model adds a complicating temporal dimension as generation output, power flows, bus phase angles, and storage levels must be calculated for each time period. As a consequence, the wall time increases significantly to 11.81 hours for the short peak scenario and 8.13 hours for the long peak scenario when solved using a single thread, and 13.35 hours and 11.43 hours using up to 16 threads. The Benders decomposition compares favorably with the wall time reduced to 1.66 hours and 1.01 hours for the short and long peak scenarios respectively. This significant reduction in solve time comes at the cost of increased modeling complexity and therefore increased development time.

Another interesting observation in this case study is that the shape of the demand curve not only affects the amount of storage that needs to be installed, but also has a significant effect on the line expansions used. All three scenarios have quite different circuit augmentation solutions.

5. Case study: 46-bus network

The 46-bus network represents the southern part of the Brazilian transmission network. This real-world test system consists of 46 buses and 79 rights of way, and has total demand of 6880MW. The tabulated data is available in [12]. We permit the installation of a maximum of 5 new or reinforcing circuits. The investment cost of the optimal expansion plan without ESS is US$154.42 million. This expansion plan installs one circuit rights of way 6-46, 19-25, 20-21, 28-30, and 31-32. Two circuits are installed on rights of way 5-6, 25-25, 29-30, and 42-43, and three circuits are installed on 26-29.

As with the previous case studies, we consider the short and long peak demand scenarios, and re-scale demand identically at each bus. The cost of ESS is specified at $2000/MWh, and the model is solved using the same computing infrastructure used in Section 4. Results are given in Table 3.

Despite the increase network size the TEP problem without storage remains easily solved approximately 6 seconds. When storage is considered solution time for the monolithic formulation solved by CPLEX 1 increases to 18.58 hours and 39.98 hours for the short and long peak scenarios respectively. For CPLEX 16 this decreases to 17.59 hours and 23.20 hours. Benders decomposition reduces wall time to 4.40 hours for the short peak scenario. However, the technique fails to improve upon the wall time for the long peak scenario compared to CPLEX 16, requiring 32.85 hours to obtain the optimal solution.

6. Discussion

The optimal solutions presented in the case studies install a large amount of very cheap energy storage in lieu of installing transmission infrastructure.

A generalized Benders decomposition approach shows some initial success in improving the solution wall time of TEP problems with storage. However, under the long peak scenario on the 46-bus test network the decomposition approach took 9.65 hours longer than solving the monolithic formulation with a commercial solver with 16 threads available.

Descriptive statistics comparing the LP subproblem solution (wall) time of each scenario is given in Table 4. The LP solution method is set to the *Automatic* setting, in which case the CPLEX chooses which optimization algorithm to use. A small number of subproblems return a non-optimal solution status code. Experience dictates that cut separation is not necessarily reliable in this case, so the LP solution method is explicitly set to the dual simplex algorithm and the problem re-solved to optimality.

The short peak scenario required 2894 subproblems to be solved with mean solution time of 4.78 seconds. There is one extreme outlier of 1.18 hours. In contrast, the long peak scenario required 12769 subproblems to be solved with a mean time of 5.53 seconds. There are a number of extreme outliers, with a maximum solution time of 42 minutes. The difference in mean solution times is not statistically significant at the 95% significance level.

The proportion of total wall time spent solving LP subproblems is 0.87 for the short peak scenario. The bulk of the remaining time is consumed by the branch and cut. For the long peak, this proportion is 0.48, which combined with the large number of subproblems solved suggests the optimality cuts generated are quite shallow. That the Benders decomposition investigates 594964 nodes whereas 5779 nodes are investigated by CPLEX 16 seems to support this.

It is likely that the observed extreme outlier LP solution times are the result of degeneracy in the LP subproblems.

It is perhaps counter-intuitive that with the exception of the long peak scenario for the 46-bus network, the parallel branch and cut confers little or no advantage over the single threaded branch and cut. A proportion of this might be explained by the additional synchronization required, which often totals more than 10% of the total wall time. In the case of the long peak scenario, the parallel branch and cut applies 100 flow cuts, 797 mixed integer rounding cuts, and 1 Gomory fractional cut, whereas the sequential branch and cut applies only 27 flow cuts, 174 mixed integer rounding cuts, and 1 Gomory fractional cut. Clearly there is something about the structure of this particular problem that makes it particularly amenable to parallelization.

7. Conclusion

In this paper we have shown how the TEP can be extended to consider ESS as a means of transmission upgrade deferral. The resulting TESP model is significantly more complicated as the time dimension has to be explicitly considered. Indeed, as our results have shown, the time dynamics of demand have a significant impact on the network design, unlike for standard TEP where only the peak load matters. The model has been tested against the well known Garver's 6-bus, IEEE 25-bus, and Brazilian 46-bus test systems under two different demand scenarios.

Our results show that installation of ESS is an effective means of transmission upgrade deferral, however storage is unlikely to be installed where circuit installation is of comparatively low cost. The amount of storage installed is found to be dependent on the demand scenario under which it is operated.

The model is computationally demanding for modestly sized test networks, but its structure makes it amenable to decomposition. Our Benders decomposition approach significantly reduces solution time in most test cases. As the technique may potentially require a large number of subproblems to be solved, the choice of LP solver can have a substantial impact on wall time.

Since the network design depends not just on the peak demand but also on the dynamics of load over time, the current model is not yet robust to the stochastic variation in demand profiles that an electricity network may experience. To deal with this, it will be necessary to consider network designs that can satisfy multiple scenarios. Our proposed approach can be expected to extend easily to such a multi-scenario extension as this simply requires multiple sub-problems to be solved, possibly in parallel, to get a set of optimality cuts to be added to the master problem. However further research into an appropriate selection of scenarios is required before this extension can be tested empirically.

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Figures

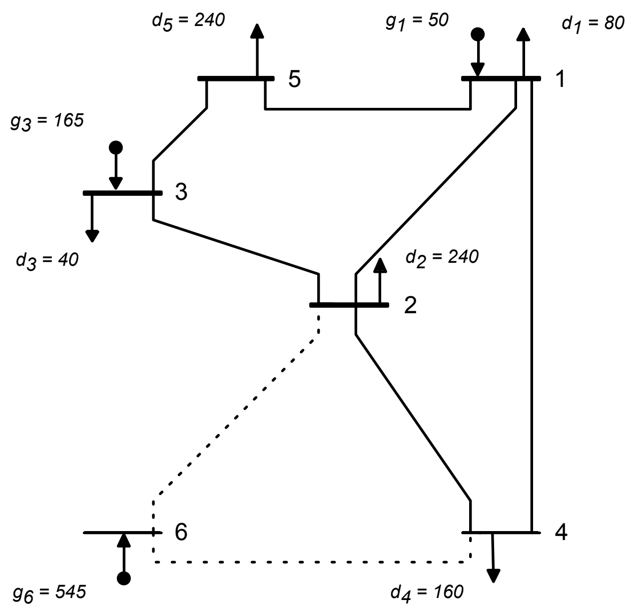

Figure 1: Initial network topology of for Garver's 6-bus test system.

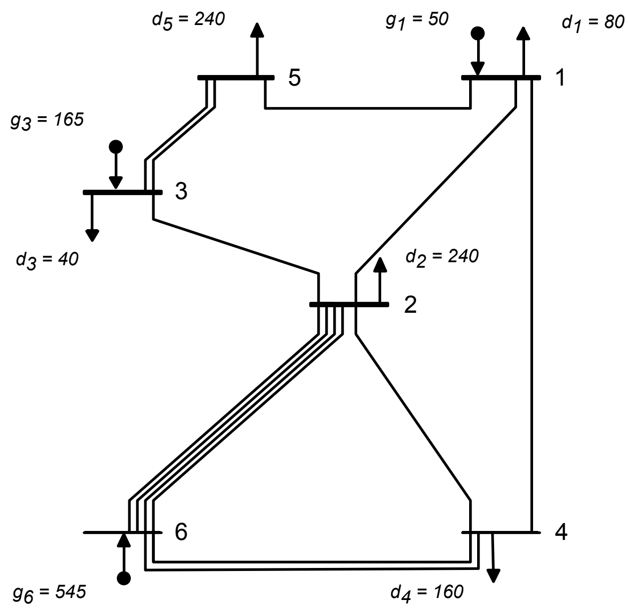

Figure 2: Optimal expansion plan without considering ESS.

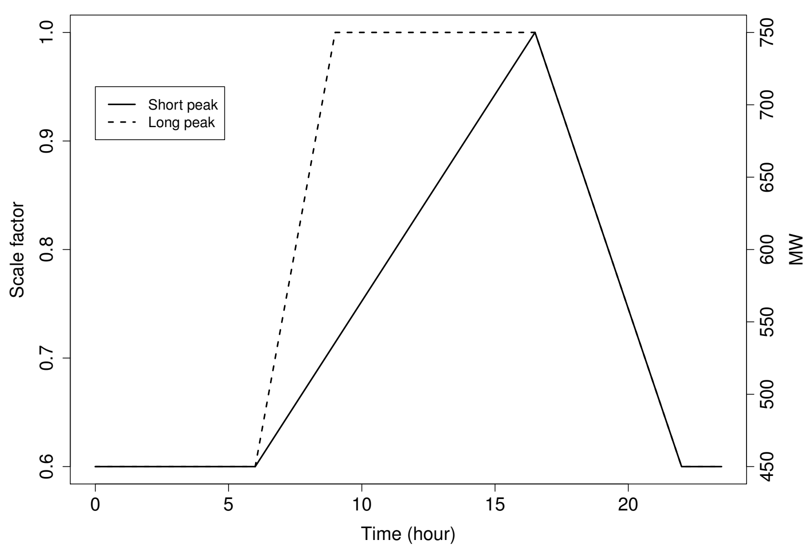

Figure 3: Demand over time for short peak and long peak scenarios. The left y-axis shows the scale factor used to re-scale maximum demand. The right y-axis shows re-scaled total demand in Garver's 6 bus test system.

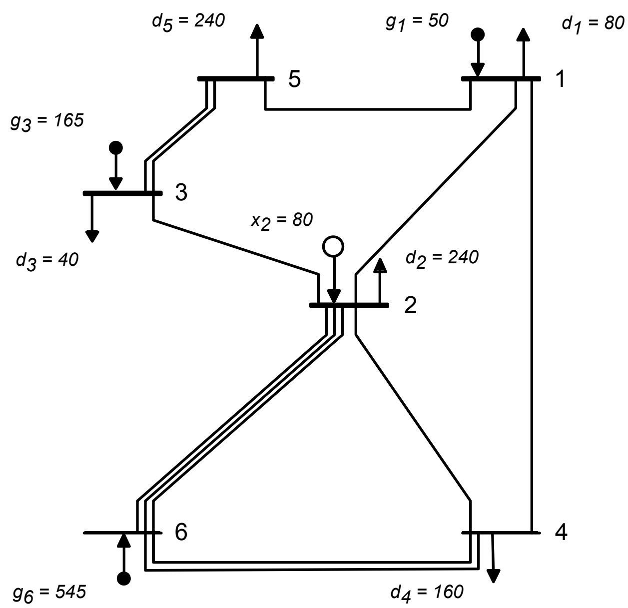

Figure 4: Optimal expansion plan considering ESS for short peak scenario.

Tables

Table 1: Maximum storage cost coefficients for Garver's 6-bus network.

Scenario	Storage Cost (US$/MWh)	Total Cost (US$$10^3$)	Circuits	Total Storage (MWh)	CPLEX Wall Time (s)	Benders Wall Time (s)
No storage	-	200.00	2-6 (4) 3-5 (1) 4-6 (2)	0	1.72	-
Short peak	370	199.55	2-6 (3) 3-5 (1) 4-6 (2)	80	1.27	1.29
Short peak	70	175.59	2-6 (3) 3-5 (1) 4-6 (2)	80	1.25	1.32
Long peak	70	198.91	2-6 (3) 3-5 (1) 4-6 (2)	413	1.46	1.52

Table 2: Results for IEEE 25-bus network.

Scenario	Storage Cost (US$/MWh)	Obj. (US$$10^3$)	Circuits	Total Storage (MWh)	CPLEX 1 (s)	CPLEX 16 (s)	Benders (s)
No storage	-	107706	1-2 (1) 7-13 (1) 8-22 (3) 12-14 (2) 12-23 (3) 13-18 (2) 13-20 (4) 24-25 (2)	0	-	3.54	-
Short peak	2000	32032	7-16 (1) 12-23 (2) 13-18 (3) 13-20 (4) 24-25 (2)	1598	42530	48042	5957
Long peak	2000	43812	5-25 (2) 7-16 (1) 8-22 (1) 12-23 (1) 13-18 (4) 13-20 (2)	2619	29254	41158	3620

Table 3: Results for 46-bus network. All durations are wall time.

Scenario	Storage Cost (US$/MWh)	Obj. (US$$10^3$)	Circuits	Total Storage (MWh)	CPLEX 1 (s)	CPLEX 16 (s)	Benders (s)
No storage	-	154420	5-6 (2) 6-46 (1) 19-25 (1) 20-21 (1) 24-25 (2) 26-29 (3) 28-30 (1) 29-30 (2) 31-32 (1) 42-43 (2)	0	-	6.08	-
Short peak	2000	72355	5-6 (2) 6-46 (1) 20-21 (2) 20-23 (1) 42-43 (1)	4596	66873	63333	15855
Long peak	2000	100111	5-6 (2) 6-46 (1) 20-21 (2) 20-23 (1) 31-32 (1) 42-43 (2) 13-20 (3)	10859	143941	83526	118262

Scenario	Total solved	Re-solved	Min (s)	Max (s)	Mean (s)	Stdev (s)	LP Wall (s)	Total Wall (s)
Short	2,894	12	0.50	4,240.93	4.78	78.78	13,833	15,855
Long	12,769	4	0.51	2,517.96	5.53	24.27	56,766	118,261

Table 4: Comparison of LP subproblem wall time for the 46-bus test system.